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Spectral Degeneracy in Supersymmetric Gluodynamics and One-Flavor QCD related to $\mathcal{N} = {}^1\!/_2$ SUSY

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Abstract

In supersymmetric gluodynamics ($\mathcal{N} = 1$ super-Yang-Mills theory) we show that the spectral functions induced by the *nonchiral* operator Tr $\left(G_{\alpha\beta}\,\bar{\lambda}^2\right)$ are fully degenerate in the $J^{PC} = 1^{\pm -}$ channels. The above operator is related to $\mathcal{N} = {}^1\!/2$ generalization of SUSY. Using the planar equivalence, this translates into the statement of degeneracy between the mesons produced from the vacuum by the operators $\left(\bar{\Psi}\vec{E}\Psi + i\bar{\Psi}\vec{B}\gamma^5\Psi\right)$ and $\left(\bar{\Psi}\vec{B}\Psi - i\bar{\Psi}\vec{E}\gamma^5\Psi\right)$ in one-flavor QCD, up to $1/N$ corrections. Here Ψ is the quark field, and $\vec{E}, \vec{B}$ are chromoelectric/chromomagnetic fields, respectively.

Recently [1, 2] a C-deformation of $\mathcal{N}=1$ superalgebra corresponding to non-anticommuting Grassmann coordinates θ which leads to the so-called $\mathcal{N}=\,^1\!/2$ superalgebra has been discussed. A C-deformed supersymmetric (SUSY) Yang-Mills theory (SUSY gluodynamics) was constructed and discussed by Seiberg [3] (for further developments concerning some nonperturbative aspects of the theory see e.g. [4]). The parameter of deformation C in Ref. [3] corresponds to the constant selfdual graviphoton background.

We will show that the very possibility of the above C-deformation is related to a spectral degeneracy in *conventional* $\mathcal{N}=1$ SUSY gluodynamics which, by virtue of the planar equivalence between SUSY and orientifold theories [5], can be readily copied in one-flavor QCD, implying that the masses and coupling constants of hybrid $1^{\pm-}$ (color singlet) mesons are degenerate. That an infinite number of spectral degeneracies must take place in the orientifold theory at $N\to\infty$ was noted in [5]. The degeneracy we will discuss here is singled out because of its connection with a special operator (of nonchiral type).

The above spectral degeneracy, in turn, suggests that $\mathcal{N}=\,^1\!/2$ supersymmetry remains valid for coordinate-dependent C parameter.

The Lagrangian of $\mathcal{N}=1$ SUSY gluodynamics is

$$\mathcal{L}=\frac{1}{2g^2}\int d^2\theta\,\mathrm{Tr}\,W^2+\mathrm{H.c.}\,, \tag{1}$$

where

$$W_\alpha=\frac{1}{8}\,\bar{D}^2\left(e^{-V}D_\alpha e^V\right)=i\left(\lambda_\alpha+i\theta_\alpha D-\theta^\beta\,G_{\alpha\beta}-i\theta^2\mathcal{D}_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}\right)\,. \tag{2}$$

and $\int d^2\theta\,\theta^2=1$.

The spectral degeneracy in supersymmetric gluodynamics follows from the fact that the n-point functions

$$\begin{aligned}&\left\langle\mathrm{Tr}\,\left(G_{\alpha\beta}\bar\lambda^2(x)\right),\,\mathrm{Tr}\,\left(G_{\gamma\rho}\bar\lambda^2(0)\right)\right\rangle_{\rm connected}=0\,,\\ &...\\ &\left\langle\mathrm{Tr}\,\left(G_{\alpha\beta}\bar\lambda^2(x_n)\right),\,\mathrm{Tr}\,\left(G_{\alpha\beta}\bar\lambda^2(x_{n-1})\right),\,...,\,\mathrm{Tr}\,\left(G_{\gamma\rho}\bar\lambda^2(0)\right)\right\rangle_{\rm connected}=0\,,\end{aligned} \tag{3}$$

for any n and x_i. Here $\langle ...\rangle$ stands for the vacuum expectation value (T product is implied on the left-hand side and elsewhere, where necessary) and

$$\bar\lambda^2\equiv\bar\lambda_{\dot\alpha}\bar\lambda^{\dot\alpha}\,.$$

Derivation of Eq. (3) is rather straightforward. To begin, let us consider the two-point function in the first line. We start from

$$\{Q_\alpha,\,\mathrm{Tr}\,\left(\lambda_\beta\,\bar\lambda^2\right)\}=\mathrm{Tr}\,\left(G_{\alpha\beta}\,\bar\lambda^2\right), \tag{4}$$

expressing the fact that the operator Tr $\left(G_{\alpha\beta}\,\bar\lambda^2\right)$ is Q-exact. Next, we replace one of the operators Tr $\left(G_{\alpha\beta}\,\bar\lambda^2\right)$ in the first line of Eq. (3). Taking into account that $Q_\alpha|\text{vac}\rangle = 0$ we then transform it as follows:

$$\left\langle \left\{ \text{Tr}\,\left(G_{\alpha\beta}\bar\lambda^2(x)\right) Q_\gamma \right\},\, \text{Tr}\,\left(\lambda_\rho\,\bar\lambda^2(0)\right)\right\rangle\,. \tag{5}$$

What remains to be done is to use the relation

$$\{G_{\alpha\beta}Q_\gamma\} \sim \mathcal{D}_{\beta\dot\gamma}\bar\lambda^{\dot\gamma}\,, \tag{6}$$

which vanishes on mass shell. In other words, the operator Tr $\left(G_{\alpha\beta}\,\bar\lambda^2\right)$ is Q-closed modulo the equation of motion. In the correlation function (5) the equation of motion $\mathcal{D}_{\beta\dot\gamma}\bar\lambda^{\dot\gamma}$ contracts the propagator $\langle\bar\lambda(x)\,\lambda(0)\rangle$ resulting in the contact term of the type

$$\delta^4(x)\,\left\langle g^2\text{Tr}\,\left(\bar\lambda^2\bar\lambda^2\right)\right\rangle \tag{7}$$

The single-trace operator Tr $\left(\bar\lambda^2\bar\lambda^2\right)$ was discussed in detail in [6], Sect. 2.1, where it was shown that its vacuum expectation value vanishes. (Fully antisymmetric in the Lorentz indices part vanishes kinematically while the symmetric part is Q-exact and thus its expectation value also vanishes [1].) This concludes the proof of the first line in Eq. (3).

For $n > 2$ we replace the right-most operator in the n-point function according to Eq. (4) and then drag Q to the left, generating *en route* various contact terms $\sim$ Tr $\left(\bar\lambda^2\bar\lambda^2\right)$ and correlators with $n-2$ operators Tr $\left(G_{\alpha\beta}\,\bar\lambda^2\right)$, for which we repeat the substitution (4), eventually arriving at zero in the right-hand side.

Equations (3) are trivial in perturbation theory. SUSY gluodynamics is thought to be confining, however, with color-singlet composite states in the spectra. At the level of color-singlet composites Eq. (3) implies a perfect spectral degeneracy of hybrid mesons with quantum numbers $J^{PC} = 1^{+-}$ and $J^{PC} = 1^{--}$, respectively, produced by the operator Tr $\left(G_{\alpha\beta}\,\bar\lambda^2\right)$, much in the same way as the vanishing of the correlators

$$\left\langle \text{Tr}\,\left(\bar\lambda^2(x_n)\right),\,\text{Tr}\,\left(\bar\lambda^2(x_{n-1})\right),\,\ldots,\,\text{Tr}\,\left(\bar\lambda^2(0)\right)\right\rangle_{\text{connected}} \tag{8}$$

in the chiral sector [7] implies a complete spectral degeneracy in the $0^{\pm +}$ channels. The easiest way to establish quantum numbers of the operator Tr $\left(G_{\alpha\beta}\,\bar\lambda^2\right)$ is to rewrite it in the Majorana notation for gluino,

$$\text{Tr}\,\left(G_{\alpha\beta}\,\bar\lambda^2\right) \to \left(\bar\lambda\vec{E}\lambda + i\bar\lambda\vec{B}\gamma^5\lambda\right) + i\left(\bar\lambda\vec{B}\lambda - i\bar\lambda\vec{E}\gamma^5\lambda\right)\,, \tag{9}$$

[1]Note that Tr $\left(\bar\lambda^2\bar\lambda^2\right)$ involves d^{abc} structure constants. One of the consequences is the fact that Tr $\left(\bar\lambda^2\bar\lambda^2\right) \equiv 0$ for the SU(2) gauge group because in SU(2) there are no d symbols.

and there is no interference between the two terms. Here $\vec{E}$ and $\vec{B}$ stand for chromoelectric and chromomagnetic fields, respectively. The spectral degeneracy in the $J^{PC} = 1^{\pm -}$ channels induced by two terms in Eq. (9) can be viewed as a manifestation of chromoelectric/magnetic duality appropriate to $\mathcal{N} = 1$.

The C-deformed $\mathcal{N} = {}^1\!/2$ SYM theory has the Lagrangian [3]

$$\mathcal{L}_C = \mathcal{L}_{C=0} + \left(\frac{1}{4g^2}\right)\left(-iC^{\alpha\beta}\mathrm{Tr}\,\left(G_{\alpha\beta}\,\bar{\lambda}^2\right) + (\mathrm{det}C)\mathrm{Tr}\,\left(\bar{\lambda}^2\bar{\lambda}^2\right)\right). \tag{10}$$

Since Q is conserved for any C, the expansion of the vacuum energy $\mathcal{E}(C)$ in the powers of C must generate zeros order by order. This leads to a set of the low-energy theorems the first of which has the form

$$i\int d^4x \left\langle \mathrm{Tr}\,\left(G_{\alpha\beta}\bar{\lambda}^2(x)\right),\, \mathrm{Tr}\,\left(G_{\gamma\rho}\bar{\lambda}^2(0)\right)\right\rangle_{\mathrm{connected}} = 0\,. \tag{11}$$

Higher order terms in C emerge with higher n-point functions. These low-energy theorems present a weak (integrated) form of (3). The validity of (3) locally, point by point, means that even if one considers C to be a coordinate-dependent function, the vacuum energy density would still vanish, which, in turn, suggests that $\mathcal{N} = {}^1\!/2$ theories with coordinate-dependent $C^{\alpha\beta}$ are Q-invariant too. Examination of the corresponding supertransformations[2] confirms this statement. In other words one can say that two supercharges Q_α remain conserved in arbitrary self-dual graviphoton background, not necessarily in coordinate-independent background. This is akin to the statement that two out of four supercharges are conserved in the arbitrary self-dual gauge-field background (e.g., instanton), which leads to the Bose-Fermi spectral degeneracy in this background, resulting in multiple exact results of the type of the NSVZ β function [8].

Now, let us turn to implications of the above-established spectral degeneracy in the non-supersymmetric orientifold theory. Using the planar equivalence [5] we can pass to orientifold theory (at large N) by mere rewriting (3) in terms of fields relevant to this theory. In the fermion sector we replace the Weyl adjoint gluino by a Dirac field Ψ in the two-index antisymmetric representation of SU(N). The gluon sector remains intact. The operator $\bar{\lambda}^2$ is then replaced by $\left(\bar{\Psi}\,(1+\gamma^5)\Psi\right)^i_j$ where i, j are color indices, and the trace $\mathrm{Tr}\,\left(G_{\alpha\beta}\bar{\lambda}^2(x)\right)$ must be replaced by

$$\left(G_{\mu\nu} - \tilde{G}_{\mu\nu}\right)^j_i\,\left(\bar{\Psi}\,(1+\gamma^5)\Psi\right)^i_j \equiv \mathcal{G}_{\mu\nu}\,. \tag{12}$$

In the orientifold theory at $N \to \infty$ Eq. (3) is then replaced by

$$\langle \mathcal{G}_{\mu\nu}(x_n),\, \mathcal{G}_{\rho\sigma}(x_{n-1})\,, \ldots, \mathcal{G}_{\phi\chi}(0)\rangle = 0\,. \tag{13}$$

[2]See Eq. (4.16) in [3]. The only C-containing term is in the supertransformation of λ, namely, $(\delta\lambda_\alpha)_C = \epsilon^\beta C_{\alpha\beta}\bar{\lambda}^2$.

The consequence is the same as in $\mathcal{N} = 1$ SUSY gluodynamics: the complete degeneracy of the meson parameters in the $1^{\pm -}$ channels. In particular, the spectral functions corresponding to the two-point functions induced by the currents

$$\left(\bar{\Psi}\vec{E}\Psi + i\bar{\Psi}\vec{B}\gamma^5\Psi\right)$$

and

$$\left(\bar{\Psi}\vec{B}\Psi - i\bar{\Psi}\vec{E}\gamma^5\Psi\right)$$

are identical.

Now, if we descend down to $N = 3$, the orientifold theory becomes one-flavor QCD [5]. In this case we predict the spectral degeneracy in the $1^{\pm -}$ channels up to corrections of the order of $O(1/N)$.

A remark is in order here regarding a straightforward generalization of the low-energy theorems analogous to (11) to SYM theories with matter. In the $\mathcal{N} = {}^1/2$ SYM theory with one additional flavor (either fundamental or adjoint) one gets an additional C-dependent contribution to the Lagrangian [9] of the form

$$\delta_C \mathcal{L}_{\text{matter}} = \begin{cases} -\frac{1}{\sqrt{2}} C^{\alpha\beta} \left(D_{\alpha\dot{\alpha}}\bar{\phi}\right) \bar{\lambda}^{\dot{\alpha}}\psi_\beta, & \text{fundamental,} \\ -\text{Tr}\, \frac{1}{\sqrt{2}} C^{\alpha\beta} \{D_{\alpha\dot{\alpha}}\bar{\phi},\, \bar{\lambda}^{\dot{\alpha}}\}\psi_\beta, & \text{adjoint,} \end{cases} \tag{14}$$

where ϕ, ψ are the scalar and spinor components of the chiral matter superfield either in the fundamental or the adjoint representation. Note that the deformation does not touch the structure of the flat directions in these theories.

In both theories we have two operators coupled to the term linear in C and a single Q-exact operator coupled to C^2. The corresponding low-energy theorems read

$$\int d^4x \langle O(x),\, O(0)\rangle_{\text{connected}} = 0\,, \tag{15}$$

where

$$O = \begin{cases} \frac{i}{2g^2} \text{Tr}\, G_{\alpha\beta}\bar{\lambda}^2(x) + \frac{1}{\sqrt{2}} \left(D_{\{\alpha\dot{\alpha}}\bar{\phi}\right) \bar{\lambda}^{\dot{\alpha}}\psi_{\beta\}}, & \text{fundamental,} \\ \frac{i}{2g^2} \text{Tr}\, G_{\alpha\beta}\bar{\lambda}^2(x) + \text{Tr}\, \frac{1}{\sqrt{2}} \{D_{\{\alpha\dot{\alpha}}\bar{\phi},\, \bar{\lambda}^{\dot{\alpha}}\}\psi_{\beta\}}, & \text{adjoint,} \end{cases} \tag{16}$$

plus similar multipoint correlators at zero momentum. If both the adjoint and fundamental flavors are added then the C-deformed $\mathcal{N} = 2$ SQCD emerges, where the corresponding low-energy theorems can be derived in a similar manner.

A question arises as to the possibility of extending the Veneziano-Yankielowicz effective Lagrangian [10] to include the low-energy theorems presented above in Eq. (11). This will certainly require an expansion of the field content of the Veneziano-Yankielowicz Lagrangian. As we see, there are nontrivial relations in the mixed

holomorphic-antiholomorphic sector, implying that additional composite fields of mixed chirality will have to be added.

Our last remark is of the literature character. Low-energy theorems for the quark-quark-gluon operators of a different Lorentz structure ($J^{PC} = 1^{-+}$ hybrids), were obtained in QCD *per SE*, with no reference to the large-N limit and supersymmetry, long ago, see Ref. [11]. The current $J_\mu = \bar{\Psi} G_{\mu\nu}\gamma_\nu \Psi$ considered in these works couples dotted and undated fermion fields, rather than the chiral fermion fields in our expressions.

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